

# Discounting

2016

Why is a bird in the hand is worth two in the bush.

# Think About It...

Why does charging interest make sense?  
Why is interest an exchange?

# Decision Time

Option A: a cost of 100 this year but a return 200 in benefits next year

Option B: a benefit of 50 this year and next year.

Both net \$100 of benefit, but which is better?

A Bit o' Math

# Math

- Use subscripts for elements in a sequence:

$B_1$   $B_2$   $B_3$  represents balance in  
years 1, 2, and 3

- Superscripts are exponents:

$$a^3 = a \times a \times a$$

# Math

- Percent means “per hundred”

$$5\% = 5/100 = 0.05$$

- Simple interest (P=principal, R=interest rate)

$$P_n = (1+R)^n \times P_0$$

# Problem

If the world is paying 3% then...

**\$915.14 TODAY**

**equals**

**\$1000 THREE YEARS FROM NOW**

The PRESENT VALUE of \$1000  
paid 3 years from now is \$915.14

# Problem

I put 1000 in the bank for a year at 5%...

$$P_1 = P_0 \times (1 + R)$$

$$1000 = P_0 \times (1 + 0.05)$$

$$P_0 = 1050$$

# Problem

If I want to have 1000 in the bank 3 years from now, how much should I deposit today if the interest rate is 3%?

$$P_3 = P_0 \times (1+R)^3$$

$$1000 = P_0 \times (1 + 0.03)^3$$

$$P_0 = \frac{1000}{(1.03)^3} = \frac{1000}{1.0927} = 915.14$$

Write an expression for how much you will have (FV, for "future value") if you put PV ("present value") dollars into an account at R percent interest for one year. Simplify the expression. What if it were N years?

$$FV = (1 + R)PV$$

$$FV = (1 + R)^N PV$$

Write an expression for how much you “have” now (PV) if you expect FV dollars N years ahead at R percent.

$$PV = \frac{FV}{(1 + R)^N}$$

# Three Problems

I've got this project.... What is

1. PV of \$500 I'll have to pay at end of one year of project.
1. PV of \$250 I'll have to pay at end of two years of project.
2. PV of \$800 I'll receive at end of third year of project.

Assume 5% discount rate

# Three Problems

	1	2	3
Pay	500	250	
Get			800
PV	$\frac{-500}{(1.05)^1} = -476.19$	$\frac{-250}{(1.05)^2} = -226.76$	$\frac{800}{(1.05)^3} = 691.07$

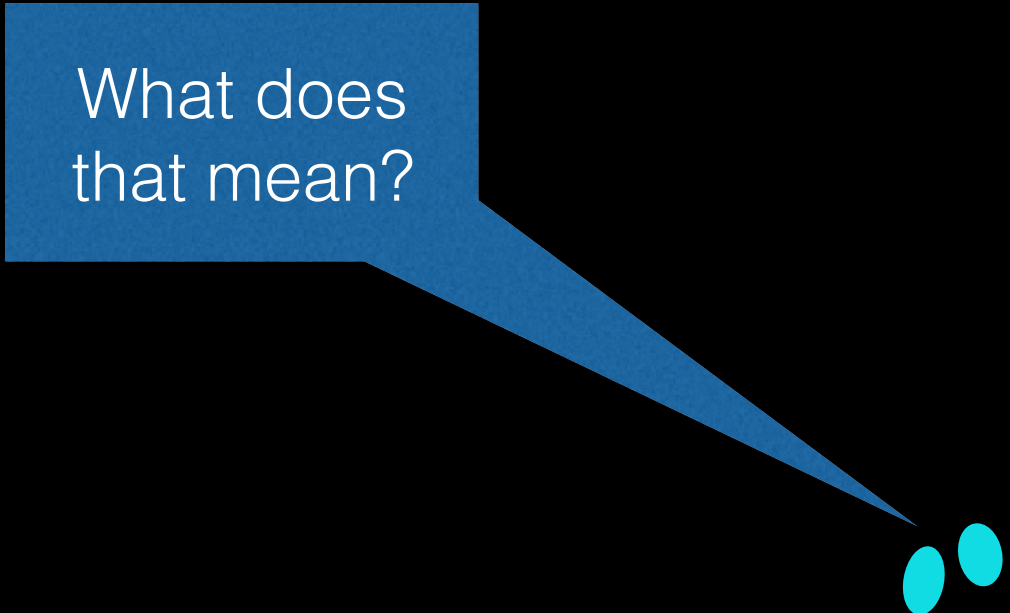
**-11.88**

Assume 5% discount rate

# Internal Rate of Return

Discount rate at which PV of project equals 0

What does  
that mean?



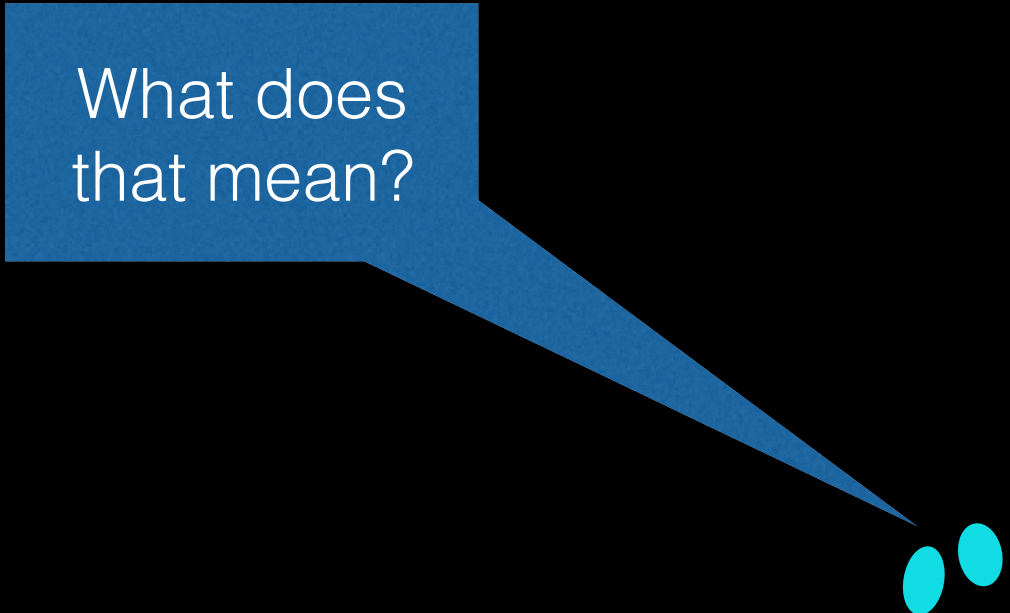
# Internal Rate of Return

1	2	3	4	5	6	7	8	9	10
-25	-20	-15	-10	0	25	25	25	25	25

$$-23.81 + -18.14 + -12.96 + -8.23 + 0 + 18.66 + 17.77 + 16.92 + 16.12 + 15.35 = 21.67$$

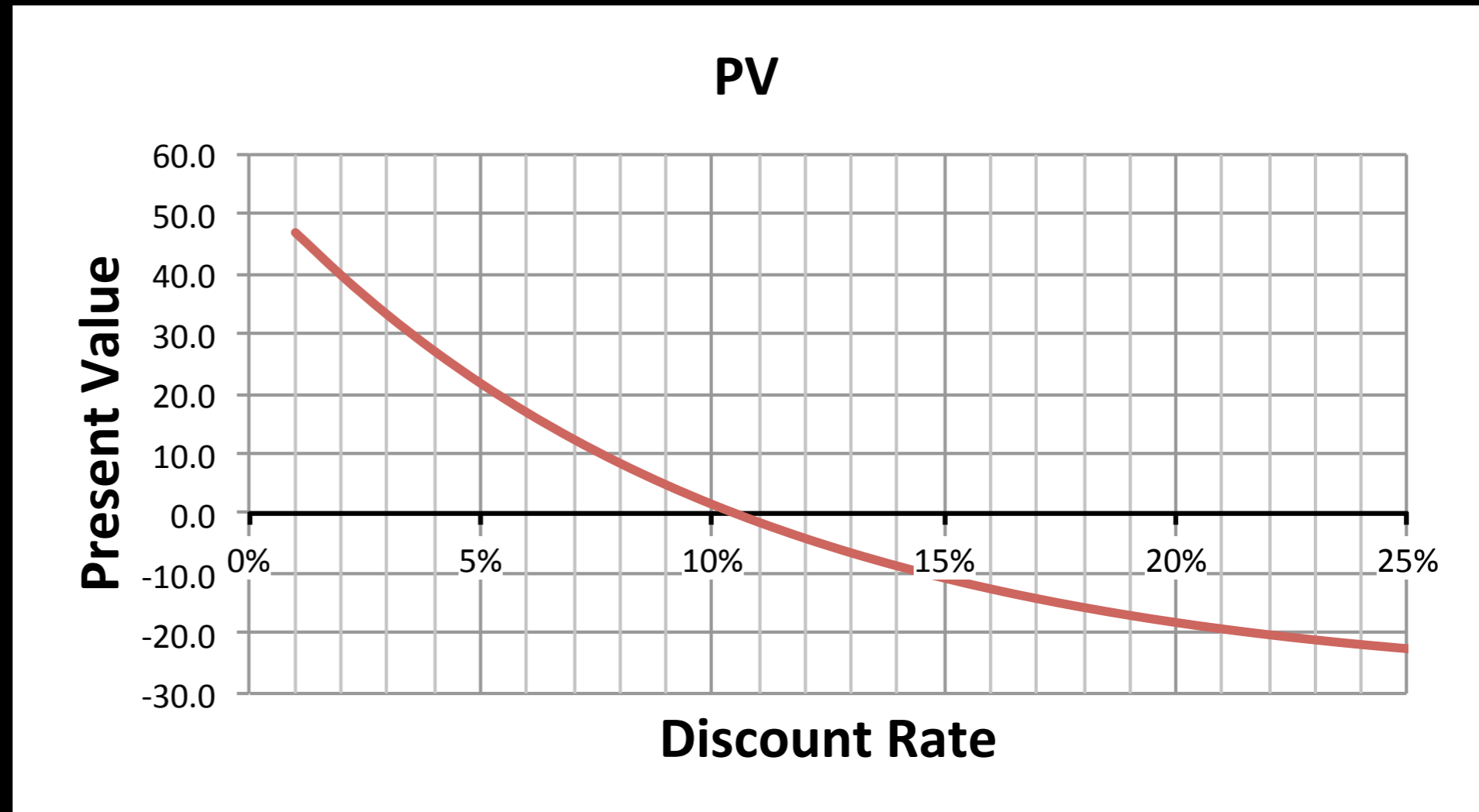
At 5% discount rate. But what about other rates?

What does  
that mean?



# Internal Rate of Return

DR	PV
1%	46.9
2%	39.6
3%	33.0
4%	27.1
5%	21.7
6%	16.8
7%	12.4
8%	8.4
9%	4.8
10%	1.5
11%	-1.5
12%	-4.2
13%	-6.6
14%	-8.8
15%	-10.8
16%	-12.6
17%	-14.2
18%	-15.7
19%	-17.0
20%	-18.2
21%	-19.3
22%	-20.2
23%	-21.1
24%	-21.9
25%	-22.5



Still don't  
get it

# Scenario

Projects cost first, benefit later.



# Basic Policy Choice

Should I do this or should I do nothing?

aka “GO/NO GO” Decision

# Basic Policy Choice

Should I do this or should I do nothing?  
aka “GO/NO GO” Decision

DECISION RULE: Do the project if the  
internal rate of return is greater than  
the discount rate

# Basic Policy Choice

Should I do project A or project B?

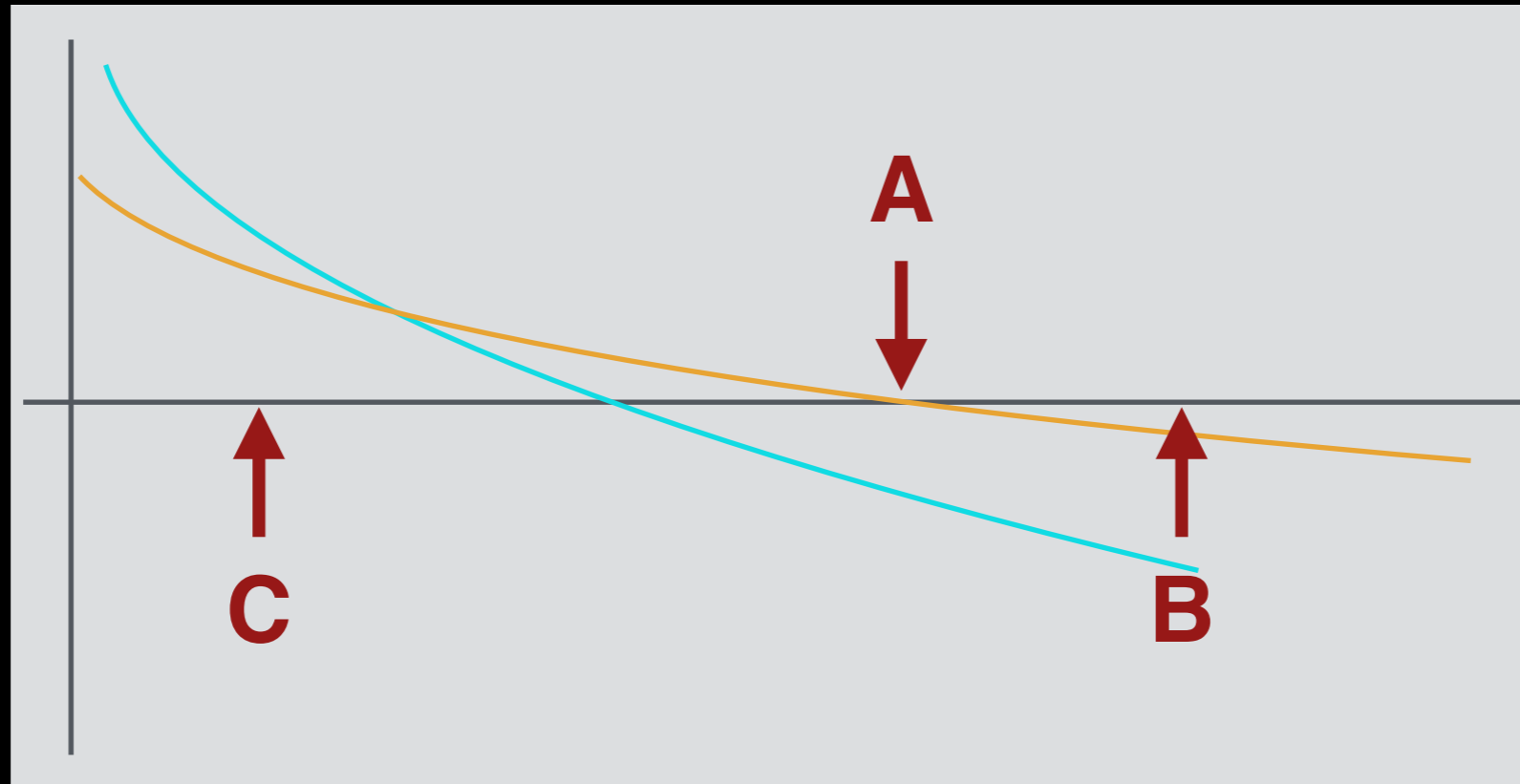
# Basic Policy Choice

Should I do project A or project B?

DECISION RULE: Choose the project  
with the higher internal rate of return

# Basic Policy Choice

## Caveats



GOLD has higher IRR (A)

but if the discount rate is C,

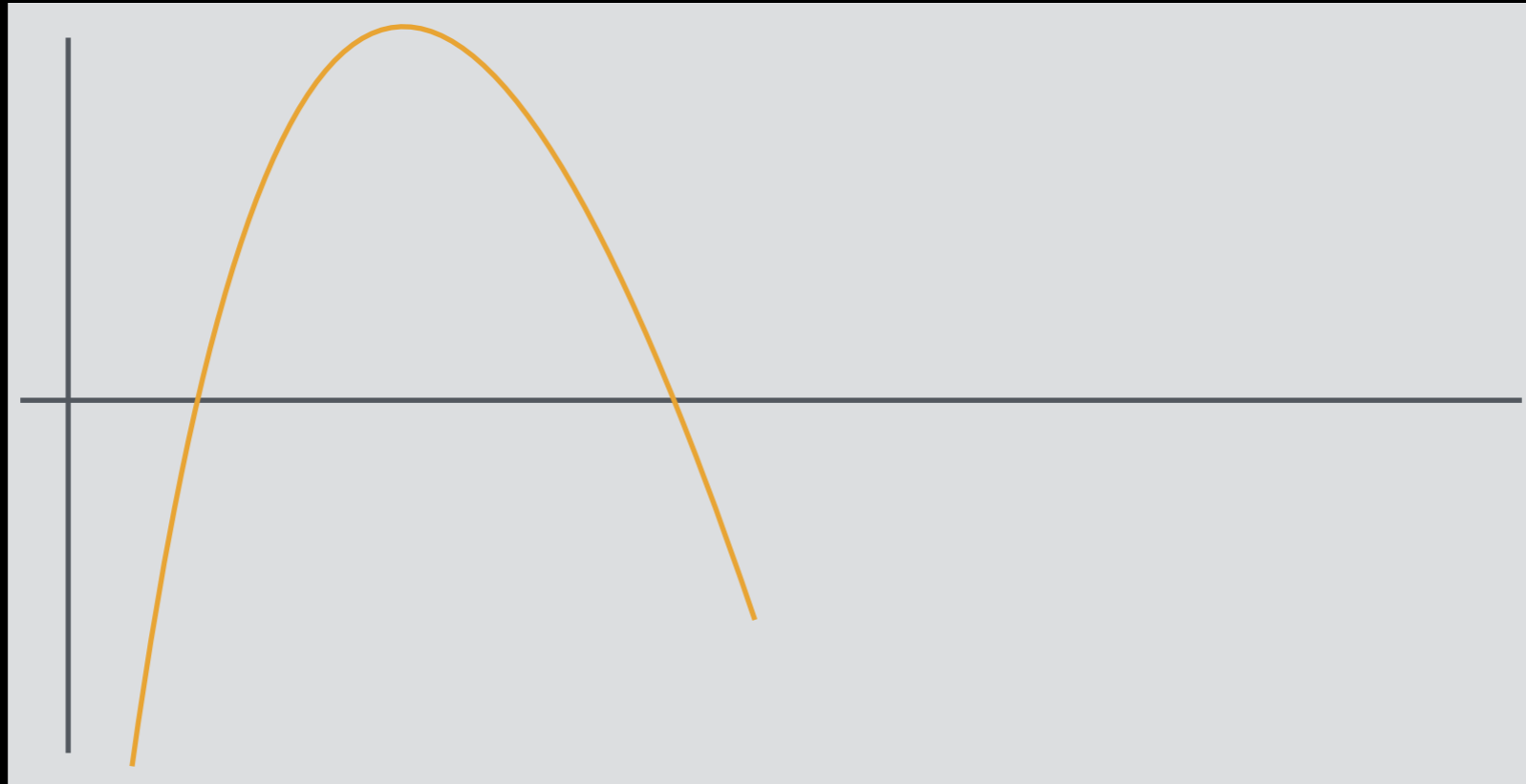
BLUE has the higher PV.

But if the discount rate is B,

neither project is better than doing nothing.

# Basic Policy Choice

## Caveats



The project has two IRRs! How?

Early costs, mid-term benefits, late costs

# Bottom Line

“Choose highest IRR” only works if

1. no budget constraint
2. projects do not preclude each other
3. streams are first negative then positive

**THUS,**

**Choose project or mix of projects with highest PV at given discount rate**

# Project Problem

A state agency is considering a childcare subsidy that would facilitate single parents' attainment of college degrees. The benefit would cost \$10k per recipient per year for four years. The expectation is that individuals with a college degree will earn more than individuals without a college degree. This means that they generate more revenue in the form of income tax. They are also less likely to require government assistance of various kinds — call this amount **A**. Assume current rules limit us to a ten year time horizon. Assume the average salary difference between non-college grads and college grads is **D** (but get the [real info here](#)) and that the marginal tax rate can be found [here](#). Assume a 5% discount rate. At first, ignore inflation.

# Payback Periods

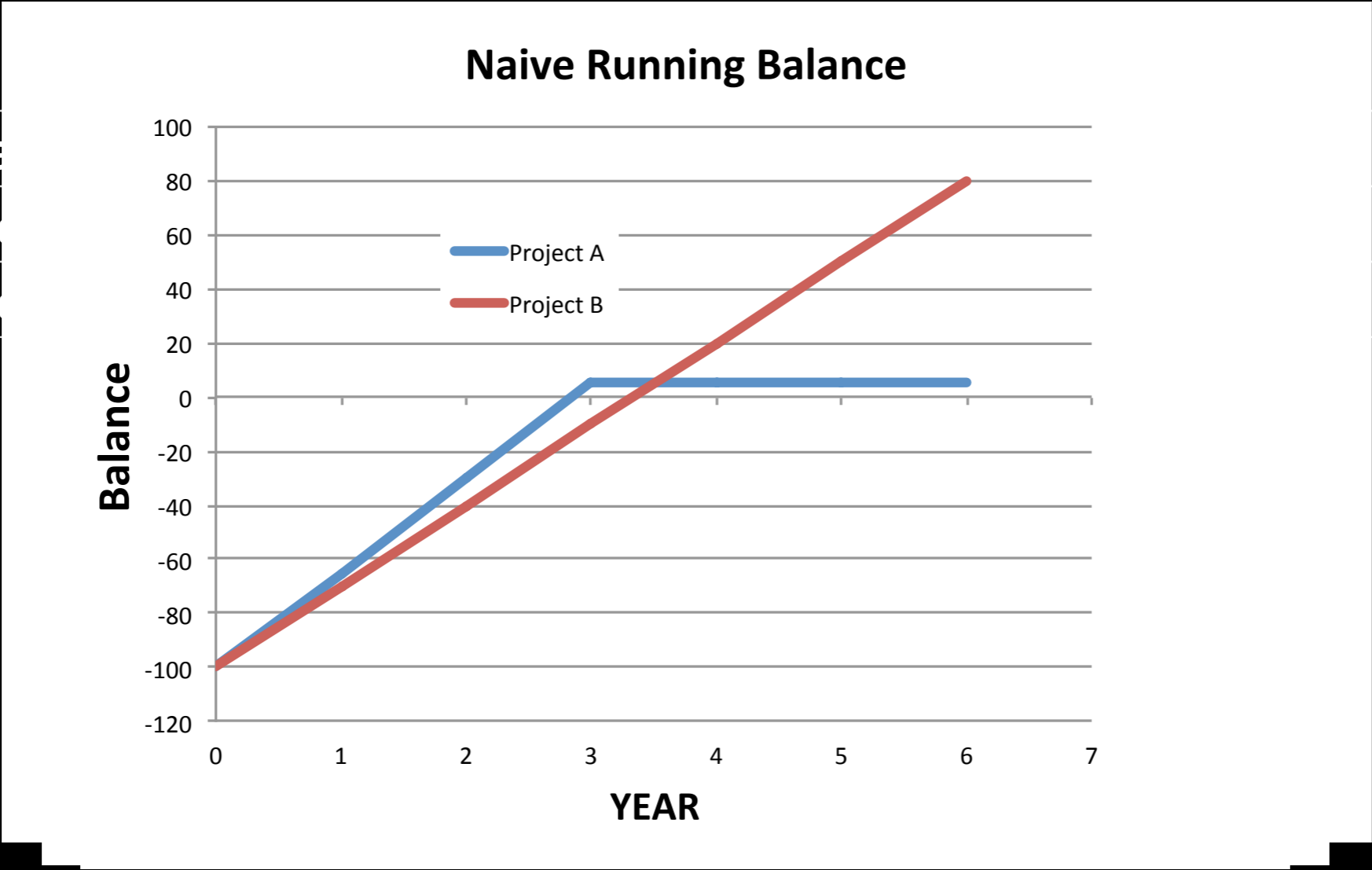
“project pays for itself in N years”

“choose project with shortest payback period”

# Payback Periods

Year	0	1	2	3	4	5	6
Project	-100	35	35	35	0	0	0
Project	-100	30	30	30	30	30	30

Year	6
Project	5
Project	80



B has higher PV across range of Drs

# Consider 2 Projects

**A:** fewer upfront costs sooner

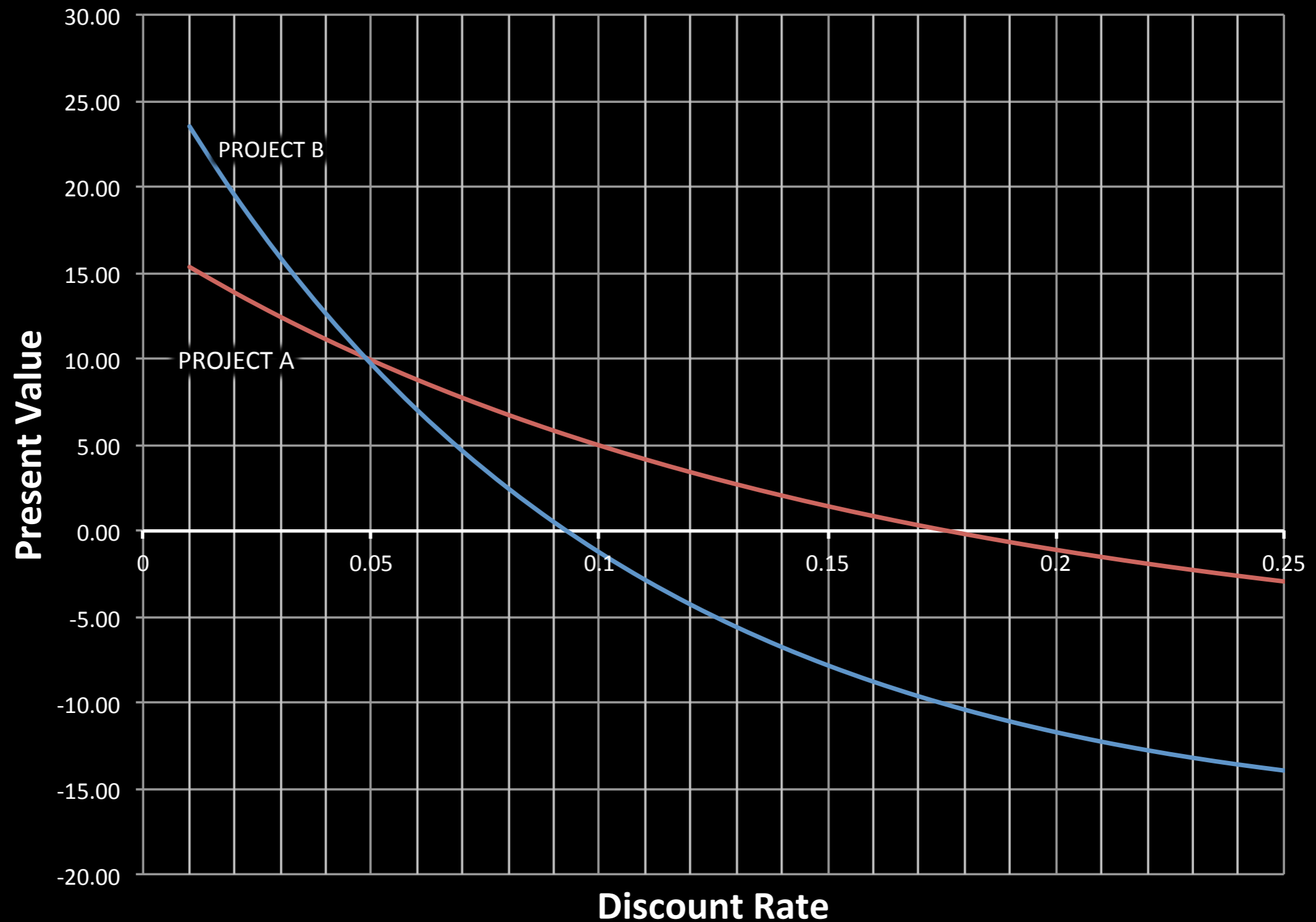
**A:** smaller revenue later

YEAR	1	2	3	4	5	6	7	8	9	10
PROJECT A	-15	-5	5	10	8	6	4	2	1	1
PROJECT B	-15	-10	-10	-5	4	4	15	15	15	15

**B:** more upfront costs later

**B:** larger revenue later

# Consider 2 Projects



# What about Inflation?

- Cost of things goes up so the value of a dollar changes over time.
- Use “deflator” to convert “nominal” \$ into YYYY \$
- Usual deflator is consumer price index (CPI)
- Look up CPI at Bureau of Labor Statistics
- Select a base year.
- Divide all CPIs by the CPI of the base year
- Divide nominal values by this number

1973	-	-	44.4	8.7	6.2
1974	-	-	49.3	12.3	11.0
1975	-	-	53.8	6.9	9.1
1976	-	-	56.9	4.9	5.8
1977	-	-	60.6	6.7	6.5
1978	-	-	65.2	9.0	5.6
1979	-	-	72.6	13.3	11.3
1980	-	-	82.4	12.5	13.6
1981	-	-	90.9	8.9	10.3
1982	-	-	96.5	3.8	6.2
1983	-	-	99.6	3.8	3.2
1984	102.9	104.9	103.9	3.9	4.1
1985	106.6	108.5	107.6	3.8	3.6
1986	109.1	110.1	109.6	1.1	1.9
1987	112.4	114.9	113.6	4.4	3.6
1988	116.8	119.7	118.3	4.4	4.1
1989	122.7	125.3	124.0	4.6	4.8
1990	128.7	132.6	130.7	6.1	5.4
1991	135.2	137.2	136.2	3.1	4.2
1992	139.2	141.4	140.3	2.9	3.0
1993	143.7	145.3	144.5	2.7	3.0
1994	147.2	149.3	148.2	2.7	2.6
1995	151.5	153.2	152.4	2.5	
1996	155.8	157.9	156.9	3.3	
1997	159.9	161.2	160.5	1.7	
1998	162.3	163.7	163.0	1.6	
1999	165.4	167.8	166.6	2.7	
2000	170.8	173.6	172.2	3.4	
2001	176.6	177.5	177.1	1.6	
2002	178.9	180.9	179.9	2.4	
2003	183.3	184.6	184.0	1.9	
2004	187.6	190.2	188.9	3.3	
2005	193.2	197.4	195.3	3.4	
2006	200.6	202.6	201.6	2.5	
2007	205.709	208.976	207.342	4.1	
2008	214.429	216.177	215.303	.1	
2009	213.139	215.935	214.537	2.7	
2010	217.535	218.576	218.056	1.5	
2011	223.598	226.280	224.939	3.0	
2012	228.850	230.338	229.594	1.7	
2013	232.366	233.548	232.957	1.5	
2014	236.384	237.088	236.736	.8	
2015	236.265	237.769	237.017	.7	
2016	-	-	-	-	-

Suppose I have 6 years of cost data and I want to express them in "constant" dollars, specifically, 2005 dollars.

	2005	2006	2007	2008	2009	2010
Data	434	435	34	32	53	76
Data	434	435	34	32	53	76
CPI	190.7	198.3	202.4	211.1	211.1	216.7
CPI/base	1	1.04	1.06	1.11	1.11	1.14
math	$\frac{434}{1}$	$\frac{435}{1.04}$	$\frac{34}{1.06}$	$\frac{32}{1.11}$	$\frac{53}{1.11}$	$\frac{76}{1.14}$
Data 2005 Dollars	434	418.3	32	28.9	47.9	66.9

\* Data not available.  
NOTE: Index applies to a month as a whole, not to any specific day.