

Problem 3: Assignments to Hospitals

The director of Burtonville Civil Defense Agency has been ordered to draw up a disaster plan for assigning casualties to hospitals in the event of a serious earthquake. For simplicity, we will assume that casualties will occur at two points in the city and will be transported to three hospitals. It is estimated that there will be 300 casualties at point A and 200 at point B. Travel times to hospitals 1, 2, and 3 are 25, 15, and 10 minutes, respectively; from point B they are 20, 5, and 15 minutes. Hospital capacities for emergency cases are 250, 150, and 150 patients. How should the victims be assigned to hospitals to minimize the total time lost in transporting them?

What is our objective here? Minimize total time lost to transport.

What quantities do we know? Transport time from point X to hospital Y is given – all told we have 6 values which we'll call T sub point–hospital:

$$T_{A1} = 25$$

$$T_{A2} = 15$$

$$T_{A3} = 10$$

$$T_{B1} = 20$$

$$T_{B2} = 5$$

$$T_{B3} = 15$$

Let the number assigned to each hospital from each point be given by X sub point-hospital. The total amount of time lost to transport will thus be:

$$TOTAL = (X_{A1} \times T_{A1}) + (X_{A2} \times T_{A2}) + (X_{A3} \times T_{A3}) + (X_{B1} \times T_{B1}) + (X_{B2} \times T_{B2}) + (X_{B3} \times T_{B3})$$

$$TOTAL = 25X_{A1} + 15X_{A2} + 10X_{A3} + 20X_{B1} + 5X_{B2} + 15X_{B3}$$

We know the total casualty estimate for each location:

$$X_{A1} + X_{A2} + X_{A3} = 300$$

$$X_{B1} + X_{B2} + X_{B3} = 200$$

And that none of the Xs (how many people are sent to a given hospital) can be negative:

$$X_{A1} \geq 0$$

$$X_{A2} \geq 0$$

$$X_{A3} \geq 0$$

$$X_{B1} \geq 0$$

$$X_{B2} \geq 0$$

$$X_{B3} \geq 0$$

Note that a more compact way to write this is to use "indexed subscripts," like this:

$$X_{ij} \geq 0, \text{ where } i=A,B \text{ and } j=1,2,3$$

or

$$X_{ij} \geq 0, \quad i \in \{A, B\}, j \in \{1, 2, 3\}$$

And we have capacity constraints at the hospitals:

$$X_{A1} + X_{B1} \leq 250$$

$$X_{A2} + X_{B2} \leq 150$$

$$X_{A3} + X_{B3} \leq 150$$